In the name of God

Producer:

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Subject:

The accpetance-rejection methods in R

For Generating the standatd normal random variable.

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Isuue:

Generate a random number with a standard normal distribution and use the rejection and acceptance algorithm method.

Solve:

First we write the algorithm in theory, then we run the algorithm in the software.

For generate a standard normal random variable such az d

We know that the standard normal distribution probability density function is:

because:

If we desire an *X* ∼ *N*(*µ,σ*2), then we can express it as *X* = *σZ* + *µ*, where *Z* denotes a random variable with the *N*(0*,*1) distribution. Thus it suffices to find an algorithm for generating *Z* ∼ *N*(0*,*1). Moreover, if we can generate from the absolute value, |*Z*| is non-negative and has density*.*

If we set this

According to the rejection and acceptance algorithm, we make the following fraction and have:

We know that the best value for the a is Maximum value of the above relationship:

According to the exponential in the above relationship we can see that the maximizes of occurs in z=1.

if we calculate it and put the values in above relationship we have:

so:

Step 1: Generate Y, an exponentioal random variable with rate 1.

Step 2: Generate a random number U.

Step 3: If , then set X=Z ;else Go to step 1.

We know that:

That:

It means that we Have two Step:

Step 1:Generate Z1 and Z2 I.I.D exponetional with rate 1.

Step 2:If ,else return to Step 1.

We know that :

by computing

Step 1: Generate *Z1,Z2* ,two exponential random variable with rate 1.

Step 2:If , set ,else go to step 1.

Step 3 Generate U a random number then:   
then .

Now we will show this solve in R program:

We repeat this alg 10^6 and we can see the mean and variance of the

> rm(list = ls())

> Z<-c()

> for(i in 1:10^6){

+ y1<-rexp(1,rate = 1)

+ y2<-rexp(1,rate = 1)

+ while(y2-((y1-1)^2)/2<=0){

+ y1<-rexp(1,rate = 1)

+ y2<-rexp(1,rate = 1)

+ }

+ y<-y2-((y1-1)^2)/2

+ u<-runif(1)

+ if(u>1/2){

+ z<- -y1

+

+ }else{

+ z<- y1

+ }

+

+ Z[i]<-z}

> mean(Z)

[1] -0.001371392

> var(Z)

[1] 0.9989004

So we can see that our Z have standard normal distrubtion with mean=0 and variance=1.

End.